Applied Academy

H-86 sector-22 Noida

MATHS CLASS X CH 8,9

Class 10 - Mathematics

Section A

1. If sin A + $\sin^2$A = 1, then the value of $\cos^2$A + $\cos^4$A is **[1]**

   a) -1 b) 2

   c) 0 d) 1

2. If sin(A + B + C) = 1, then tan (A - B) = $\frac{1}{\sqrt{3}}$ and sec(A + C) = 2, find A, B and C respectively when they are acute. **[1]**

   a) 60^o, 0^o, 30^o b) 0^o, 60^o, 30^o

   c) 60^o, 30^o, 0^o d) 30^o, 60^o 90^o

3. If $\cos\theta = \frac{3}{7}$, then $\frac{\cos\theta}{1-\sin^2\theta}$ is equal to: **[1]**

   a) $\frac{3}{\sqrt{40}}$ b) $\frac{7}{\sqrt{40}}$

   c) $\frac{3}{7}$ d) $\frac{7}{3}$

4. Which of the following is true for all values of $\theta\,(0^\circ \leq \theta \leq 90^\circ)$? **[1]**

   a) $\text{cosec}^2\theta$ - $\sec^2\theta$ = 1 b) $\cos^2\theta$ - $\sin^2\theta$ = 1

   c) $\cot^2\theta$ - $\tan^2\theta$ = 1 d) $\sec^2\theta$ - $\tan^2\theta$ = 1

5. ($\cos^4$A - $\sin^4$A) on simplification, gives **[1]**

   a) 2 $\cos^2$A - 1 b) 2 $\sin^2$A + 1

   c) 2 $\sin^2$A - 1 d) 2 $\cos^2$A + 1

6. An adult and a minor boy, standing on the ground, are 4 meters apart. The height of the adult is 4 times the height of the minor boy. If at the mid-point of the line segment joining their feet, the angles of elevation of their tops are complementary, then the height of the minor boy is **[1]**

   a) 1.5 m b) 1.3 m

   c) 1 m d) 1.2 m

7. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. The length of the string, assuming that there is no slack in the string is **[1]**

   a) $90\sqrt{3}$ m b) $60\sqrt{3}$ m

   c) 90 m d) 45 m

8. **Assertion (A):** $\sin^2\theta$ = 1, then θ = 90^o **[1]**

Reason (R): As sec $60^o = 2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

9. **Assertion (A):** For $0 < \theta \leq 90^o$, cosec θ - cot θ and cosec θ + cot θ are reciprocal of each other. **[1]**

Reason (R): $\text{cosec}^2\theta - \cot^2\theta = 1$.

a) Both A and R are true and R is the correct explanation of A.

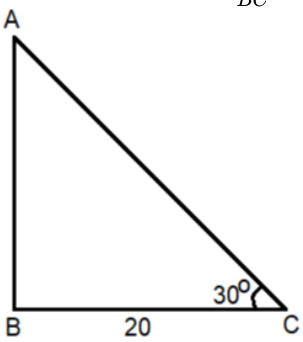
b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

10. **Assertion (A):** In the given fig if BC = 20 m then height AB is also 20 m **[1]**

Reason (R): $\tan\theta = \frac{AB}{BC}$, where θ is angle $\angle$ACB



a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

11. **Assertion (A):** The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level. **[1]**

Reason (R): The angle of depression. of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

12. In $\triangle$ABC, right-angled at A, if AB = 7 cm and AC = 24 cm, then find sin B and tan C. **[1]**

13. What is the value of $9\cot^2\theta - 9cosec^2\theta$? **[1]**

14. If secθsinθ= 0, then find the value of θ. **[1]**

15. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30^o. Calculate the distance covered by the artist in climbing to the top of the pole. **[1]**

16. A pole 6 m high casts a shadow $2\sqrt{3}$ long on the ground, then find the Sun's elevation. **[1]**

Section B

17. Prove that $\frac{(1+\sin\theta)}{(1-\sin\theta)} = (\sec\theta + \tan\theta)^2$ **[2]**

18. In a right $\triangle$ABC, right-angled at B, if tanA = 1, then verify that 2sinA·cosA = 1 **[2]**

19. Show that: $\frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)} = 1$ **[2]**

20. prove that $(\sqrt{3}+1)(3-\cot 30^\circ) = \tan^3 60^\circ - 2\sin 60^\circ$ **[2]**

21. Prove that: $\frac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1} = \text{cosec}\theta + \cot\theta$ **[2]**

22. A vertical pole stands on the level ground. From a point on the ground, 25 m away from the foot of the pole, the angle of elevation of its top is found to be 60°. Find the height of the pole. [Take $\sqrt{3}$ = 1.732.] **[2]**

23. A tower stands near an airport. The angle of elevation θ of the tower from a point on the ground is such that its tangent is $\frac{5}{12}$. Find the height of the tower, if the distance of the observer from the tower is 120 m. **[2]**

Section C

24. If sin θ + cos θ = p and sec θ + cosec θ = q, show that q(p^2 - 1) = 2p **[3]**

25. If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$ **[3]**

26. Prove that: $(cosec\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{\tan\theta+\cot\theta}$ **[3]**

27. In a right triangle ABC right angled at B if $\sin A = \frac{3}{5}$ find all the six trigonometric ratios of $\angle C$. **[3]**

28. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ. **[3]**

29. If tan A = 2, evaluate sec A sin A + tan^{2}A - cosec A **[3]**

30. Prove that $\frac{\tan\theta}{1-\tan\theta} - \frac{\cot\theta}{1-\cot\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$. **[3]**

31. If sinθ + 2 cosθ = 1 prove that 2 sinθ - cosθ = 2. **[3]**

32. In figure, $\triangle$PQR right angled at Q, PQ = 6 cm, PR = 12 cm, Determine $\angle QPR$ and $\angle PRQ$. **[3]**

33. If sec $\alpha = \frac{5}{4}$ evaluate$\frac{1-\tan\alpha}{1+\tan\alpha}$, **[3]**

34. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane. **[3]**

35. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}m$, find the speed in km/hr of the plane. **[3]**

36. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point B which is at 'b' meters above A is β. **[3]**
Prove that the height of the tower is b tan α cot β.

37. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the ground level such that $\tan\theta = 15/8$ then find the height of the kite from the ground. Assume that there is no slack in the string. **[3]**

38. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river. **[3]**

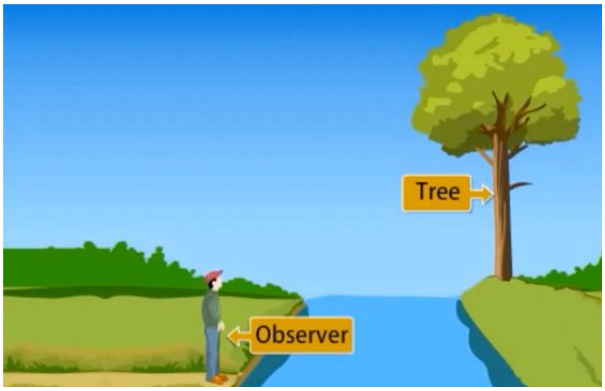
Section D

Question No. 39 to 42 are based on the given text. Read the text carefully and answer the questions: **[4]**

Akshat studies in DAV Public school,Vasant Kunj, Delhi. During summer vacation he went to his native place in a village. His grandfather took him to the bank of a nearby river. Akshat was very happy to see the pollution free environment near the river.

He was standing on the bank of the river He thought to measure the width of the river. He found that the angle of elevation of the top of a tree standing on the opposite bank was 60° When he moved 30 m away from the river the

angle of elevation reduced to 30^o

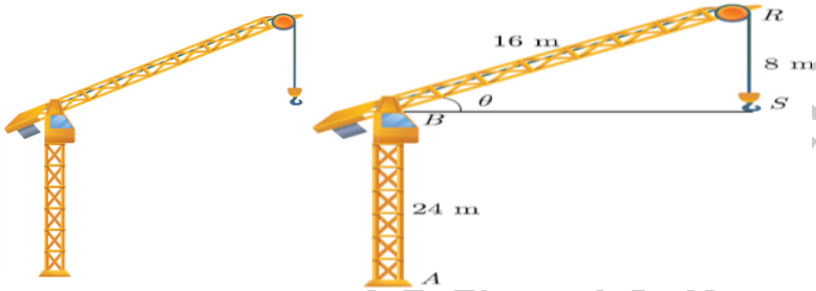


39. What is the height of the tree?
40. What is the width of the river?
41. After moving how much distance away from the river the angle of elevation becomes 45^o.
42. If the width of the river were 40 m then what would be the height of the tree selected?

Question No. 43 to 46 are based on the given text. Read the text carefully and answer the questions: **[4]**

Tower cranes are a common fixture at any major construction site. They're pretty hard to miss - they often rise hundreds of feet into the air, and can reach out just as far. The construction crew uses the tower crane to lift steel, concrete, large tools like acetylene torches and generators, and a wide variety of other building materials.

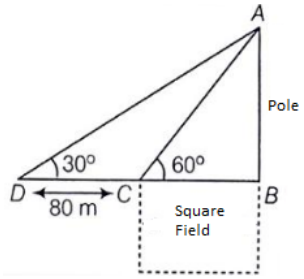
A crane stands on a level ground. It is represented by a tower AB, of height 24 m and a jib BR. The jib is of length 16 m and can rotate in a vertical plane about B. A vertical cable, RS, carries a load S. The diagram shows current position of the jib, cable and load.



43. What is the distance BS?
44. What is the angle that the jib, BR, makes with the horizontal?
45. What is the distance between the points A and S?
46. Find the angle that the crane makes with the line AS.

Question No. 47 to 50 are based on the given text. Read the text carefully and answer the questions: **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^o. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^o.



47. Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

48. Find the length of his square field so that he can buy material to do the fencing work accordingly.

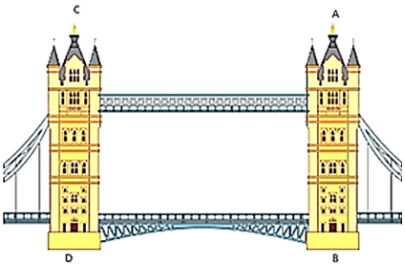
49. Find the Distance from Farmer at position C and top of the pole?

50. Find the Distance from Farmer at position D and top of the pole?

Question No. 51 to 54 are based on the given text. Read the text carefully and answer the questions: **[4]**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^o and 30^o respectively.



51. Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

52. Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

53. Find the distance between Neeta and top of tower AB?

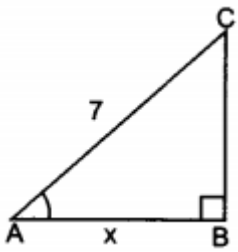
54. Find the distance between Neeta and top tower CD?

Section E

55. Prove the following identity: $\frac{\sin A}{\sec A+\tan A-1}+\frac{\cos A}{cosecA+\cot A-1}=1$ **[5]**

56. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$ or not. **[5]**

57. In $\triangle$ABC, AB = x units, AC = 7 units, and $\angle B=90^\circ$, cos B = 0. Evaluate : $\sqrt{7-x}$ tan C + $\sqrt{7+x}$ cot A - 14 cos A + 21 sin C + $\sqrt{49+x^2}$ cos B. **[5]**



58. Prove that: $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$. **[5]**

59. If cosec A - cot A = q, show that $\frac{q^2-1}{q^2+1}+\cos A=0$ **[5]**

60. If a sin θ + b cos θ = c, then prove that a cos θ – b sinθ = $\sqrt{a^2+b^2-c^2}$. **[5]**

61. Prove that : $(\sin A + \sec A)^2 + (\cos A + \text{cosec } A)^2 = (1 + \sec A \text{ cosec } A)^2$. **[5]**

62. Prove that : $(\sin\theta + 1 + \cos\theta)(\sin\theta - 1 + \cos\theta) \cdot \sec\theta \text{ cosec } \theta = 2$ **[5]**

63. Evaluate : $\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2 \tan 45° \cos^2 0° \sin 90°$. **[5]**

64. Evaluate: $4\left(\sin^4 60^\circ+\cos^4 30^\circ\right)-3\left(\tan^2 60^\circ-\tan^2 45^\circ\right)+5\cos^2 45^\circ$ **[5]**

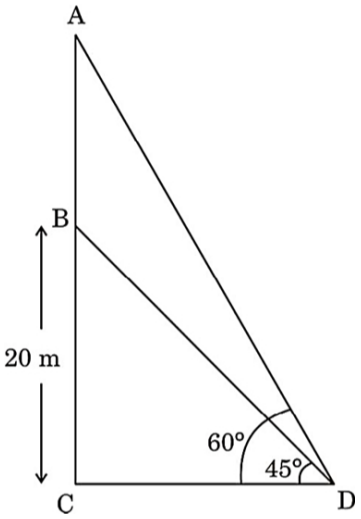
65. If cosec θ + cot θ = p, then prove that cos $\theta = \frac{p^2-1}{p^2+1}$. **[5]**

66. Prove that: $\frac{\sec A-\tan A}{\sec A+\tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$ **[5]**

67. If cos A - sin A = m and cos A + sin A = n. Show that: $\frac{m^2-n^2}{m^2+n^2}$ = -2 sin A. cos A $= -\frac{2}{\tan A+\cot A}$ **[5]**

68. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63°. Find the height of the tower and the distance of the first position from the tower. [Take tan 32° = 0.6248 and tan 63° = 1.9626] **[5]**

69. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45^o and 60^o respectively as shown in Figure. Find the height of the transmission tower. **[5]**



70. From the top of a building, 60 m high, the angle of depression of the top of a tower is 45° and from the foot of the tower, the angle of elevation of the top of the building is 60°. Find the height of the tower and its distance from the building. **[5]**

71. A tower is $100\sqrt{3}$ meters high. Find the angle of elevation if its top from a point 100 metres away from its foot. **[5]**

72. A man on the deck of a ship, 16 m above water level, observes that the angles of elevation and depression respectively of the top and bottom of a cliff are 60° and 30°.Calculate the distance of the cliff from the ship and height of the cliff.[Take $\sqrt{3} = 1.732$] **[5]**

73. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 2.5m from the banks, find the width of the river. [Take $\sqrt{3}$ = 1.732.] **[5]**

74. From a point O on the ground, the angle of elevation of the top of a tower is $30^\circ$and that of the top of the flagstaff on the top of the tower is 60°. If the length of the flagstaff is 5 metres, find the height of the tower. **[5]**

75. A boy standing on a horizontal plane finds a bird flying at a distance of 100m from him at an elevation of 30°. A girl standing on the roof of a 20-m-high building, finds the angle of elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. [Given $\sqrt{2}$=141.] **[5]**