Applied Academy

H-76 Sec 22

TRIGONOMETRY

Class 10 - Mathematics

Section A

1. If a cos θ + b sin θ = m and a sin θ - b cos θ = n, then $a^2 + b^2$ = **[1]**

   a) $n^2 - m^2$ b) $m^2 + n^2$
   c) $m^2 - n^2$ d) m^2n^2

2. If x = 4 sin θ, y = 4 cos θ, then the value of $(x^2 + y^2)$ is: **[1]**

   a) $\frac{1}{16}$ b) 16
   c) $\frac{1}{4}$ d) 4

3. If 8 tan x = 15, then sin x - cos x is equal to **[1]**

   a) $\frac{17}{7}$ b) $\frac{8}{17}$
   c) $\frac{7}{17}$ d) $\frac{1}{17}$

4. If cosec A = $\frac{7}{5}$, then value of tan A·cos A is: **[1]**

   a) $\frac{2\sqrt{6}}{5}$ b) $\frac{24}{49}$
   c) $\frac{5}{7}$ d) $\frac{7}{5}$

5. If $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$, then $\tan(\theta + \phi)$ is: **[1]**

   a) 1 b) $\frac{1}{\sqrt{3}}$
   c) $\sqrt{3}$ d) not defined

6. If sin x + $\sin^2$x = 1, then $\cos^8$x + $2\cos^6$x + cso^4 x = ________. **[1]**

   a) 0 b) 2
   c) -1 d) 1

7. If 5 cot θ = 3, then $\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}$ is equal to **[1]**

   a) $\frac{11}{18}$ b) $\frac{14}{27}$
   c) $\frac{16}{29}$ d) $\frac{16}{26}$

8. If tan θ =$\sqrt{3}$, then sec θ = **[1]**

   a) $\sqrt{\frac{3}{2}}$ b) 2
   c) $\frac{2}{\sqrt{3}}$ d) $\frac{1}{\sqrt{3}}$

9. $3\cos^2 60^o + 2\cot^2 30^o - 5\sin^2 45^o$ = ? **[1]**

   a) $\frac{17}{4}$ b) 4

c) $\frac{13}{6}$

d) 1

10. $2\cos^2\theta(1 + \tan^2\theta)$ is equal to: **[1]**

a) 3

b) 2

c) 1

d) 0

Section B

11. **Assertion (A):** $\tan^2\theta + \cot^2\theta = 2$ if $\theta = 45^o$ **[1]**

Reason (R): $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

12. **Assertion (A):** If $\sin\theta = \cos\theta$ then $\tan\theta = 1$ **[1]**

Reason (R): We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$, so $\tan\theta = 1$, as $\sin\theta = \cos\theta$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

13. **Assertion (A):** For $0 < \theta \leq 90^o$, cosec θ - cot θ and cosec θ + cot θ are reciprocal of each other. **[1]**

Reason (R): $\text{cosec}^2\theta - \cot^2\theta = 1$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

14. **Assertion (A):** In a right-angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units. **[1]**

Reason (R): (greatest side)^2 = (hypotenuse)^2 = (perpendicular)^2 + (base)^2

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

15. **Assertion (A):** $\sin^2\theta = 1$, then $\theta = 90^o$ **[1]**

Reason (R): As $\sec 60^o = 2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section C

16. Evaluate: $\tan^2 30° + \tan^2 60° + \tan^2 45°$. **[1]**

17. Prove that: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$ **[1]**

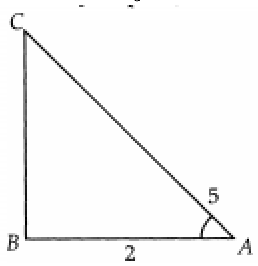
18. If $\sec\theta\sin\theta = 0$, then find the value of θ. **[1]**

19. Convert the given trigonometric equation in the simplest form. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$ **[1]**

20. If $2\cos\theta = \sqrt{3}$, then find the value of θ. **[1]**

21. Evaluate cos 30° cos 45° - sin 30° sin 45° in the simplest form **[1]**

22. If $\cos A = \frac{2}{5}$, find the value of 4 + 4$\tan^2$A. **[1]**



23. Find the value of cos60°cos45° - sin60° sin45°. **[1]**

24. Prove the trigonometric identity: cosecθ(1+ cosθ)(cosecθ - cotθ) = 1. **[1]**

25. Prove the trigonometric identity: $\sin^2\theta + \frac{1}{(1+\tan^2\theta)} = 1$ **[1]**

26. Prove $\frac{\sin\theta - 2\sin^3\theta}{2\cos^2\theta - \cos\theta} = \tan\theta$, where the angles involved are acute angles for which the expressions are defined. **[2]**

27. If tan$\theta = \frac{20}{21}$, show that $\frac{(1-\sin\theta+\cos\theta)}{(1+\sin\theta+\cos\theta)} = \frac{3}{7}$. **[2]**

28. Prove that: $\frac{\cos^2\theta}{\sin\theta} - \cos ec\theta + \sin\theta = 0$ **[2]**

29. If $x = a\cos^3\theta, y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ **[2]**

30. If cot $\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$. **[2]**

31. Evaluate: 2($\sin^2 45^o$ + $\cot^2 30^o$) - 6($\cos^2 45^o$ - $\tan^2 30^o$) **[2]**

32. Prove the trigonometric identity: $(cosec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$ **[2]**

33. Prove that: $\sec^4$A (1 - $\sin^4$A) - 2$\tan^2$A = 1 **[2]**

34. If x = a $\cos^3\theta$, y = b $\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$ = 1 **[2]**

35. If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}, 0 < \theta < 90^\circ$, find the values of $\cos\theta$ and $\tan\theta$ **[2]**

36. Prove the trigonometric identity : $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ = 2 cosec θ. **[2]**

37. Prove the identity, where the angles involved are acute angles for which the expressions are defined: **[2]**
$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

38. Prove the trigonometric identity: $(1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta) = \left(\frac{\sec\theta}{cosec^2\theta} - \frac{cosec\theta}{\sec^2\theta}\right)$ **[2]**

39. Verify that if $\tan^2\theta$ + sin θ = $\cos^2\theta$ is an identity or not. **[2]**

40. Show that: $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$ for $0^\circ \leq \theta \geq 90^\circ$ **[2]**

41. If $cosecA = \sqrt{10}$ find other five trigonometric ratios. **[3]**

42. The altitude AD of ΔABC, in which $\angle A$ is obtuse and, AD = 10 cm If BD = 10 cm and CD =10$\sqrt{3}$cm, determine $\angle A$. **[3]**

43. Prove the trigonometric identity: $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ **[3]**

44. If $\tan\theta = \frac{15}{8}$, find the value of all T-ratios of θ. **[3]**

45. If $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)$ = 1 and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)$ = 1, prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2$ **[3]**

46. If cos $\theta = \frac{3}{5}$, show that $\frac{(\sin\theta - \cot\theta)}{2\tan\theta} = \frac{3}{160}$. **[3]**

47. If $\frac{\cos\alpha}{\cos\beta}$ = m and $\frac{\cos\alpha}{\sin\beta}$ = n show that (m^2 + n^2) $\cos^2\beta$= n^2 **[3]**

48. If $\tan\theta = \frac{20}{21}$, show that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$. **[3]**

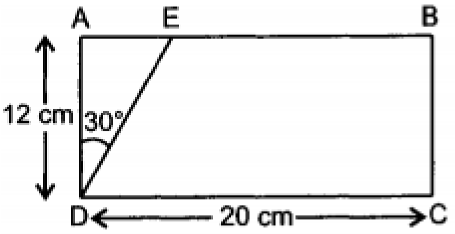
49. Evaluate: $2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$ **[3]**

50. If cosecθ = 2, show that $\left(\cot\theta + \frac{\sin\theta}{1+\cos\theta}\right)$ = 2. **[3]**

51. If $cosec\ \theta = \frac{13}{12}$ find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$ **[3]**

52. If cosB $= \frac{1}{3}$ find the other five trigonometric ratios. **[3]**

53. In the given figure, ABCD is a rectangle with AD = 12 cm and DC = 20 cm. Line segment DE is drawn making an angle of 30° with AD, intersecting AB at E. Find the length of DE and AE. **[3]**



54. If $\sin(A-B)=\frac{1}{2}, \cos(A+B)=\frac{1}{2}, 0^o < A+B \leqslant 90^o, A>B$, find the values of A and B. **[3]**

55. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$. **[3]**

56. Evaluate : $\sin^2 30° \cos^2 45° + 4\tan^2 30° + \frac{1}{2}\sin^2 90° - 2\cos^2 90° + \frac{1}{24}$. **[5]**

57. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not. **[5]**

58. If $\sec\theta = \frac{13}{5}$, show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$. **[5]**

59. Find the value of θ, if $\frac{cos\theta}{1-sin\theta} + \frac{cos\theta}{1+sin\theta} = 4, \theta \leqslant 90^o$. **[5]**

60. Prove that $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta-\csc^3\theta)} = \sin^2\theta\cos^2\theta$. **[5]**

61. Evaluate the following: $\frac{2\cos^2 60°+3\sec^2 30°-2\tan^2 45°}{\sin^2 30°+\cos^2 45°}$. **[5]**

62. If cosec A = 2, find the value of $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$. **[5]**

63. If $\sec\theta = x + \frac{1}{4x}$, $x \neq 0$, find $(\sec\theta + \tan\theta)$. **[5]**

64. If $\sin A = \frac{8}{17}$, find other trigonometric ratios of $\angle A$. **[5]**

65. Prove that: $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ = 1 + tanA + cotA = 1 + secA cosecA **[5]**

66. If cosec A - cot A = q, show that $\frac{q^2-1}{q^2+1} + \cos A = 0$ **[5]**

67. If $\sec\theta = \frac{5}{4}$, verify that $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\sin\theta}{\sec\theta}$. **[5]**

68. If $x = \gamma\cos\alpha\sin\beta$; $y = \gamma\cos\alpha\cos\beta$ and $z = \gamma\sin\alpha$, show that $x^2 + y^2 + z^2 = \gamma^2$. **[5]**

69. If $\tan\theta = \frac{a}{b}$, find the value of $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$. **[5]**

70. Prove that: $\frac{\sec A-\tan A}{\sec A+\tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$ **[5]**

71. Evaluate: $\sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 90°$ **[5]**

72. Prove the trigonometric identity: **[5]**

$$2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$$

73. In a $\triangle$ABC, $\angle$B = 90°, AB = 5 cm and (BC + AC) = 25 cm. Find the values of sin A, cos A, cosec C and sec C. **[5]**

74. Prove that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$. **[5]**

75. If cosec θ + cot θ = p, then prove that cos θ = $\frac{p^2-1}{p^2+1}$. **[5]**